Exceptional point contours in periodic unitary and thermal dynamics

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Introduction

We investigated the dynamics of a qubit undergoing periodic evolution on a two-dimensional temporal space, that is part thermal and part unitary. Properties of such evolution can be studied via Non-Hermitian Floquet Hamiltonians that can be in the $\mathcal{PT}$-symmetric or $\mathcal{PT}$-broken phases. We map out the phase diagram of the system, along with the exceptional point (EP) contours through analytical and numerical methods. We analyse the system analytically for N-cycles to observe the stroboscopic behaviour through the effective Floquet Hamiltonian. Our results suggest that dynamics in unitary and thermal systems are a new avenue to realize EP degeneracies.

Model

$$H_1(t) = J(t)\sigma_x \qquad H_2(t) = \iota\gamma(t)\sigma_z \tag{1}$$

This is our Non-Hermitian Floquet Hamiltonian where $J(t)$ and $\gamma(t)$ are arbitrary functions of time.

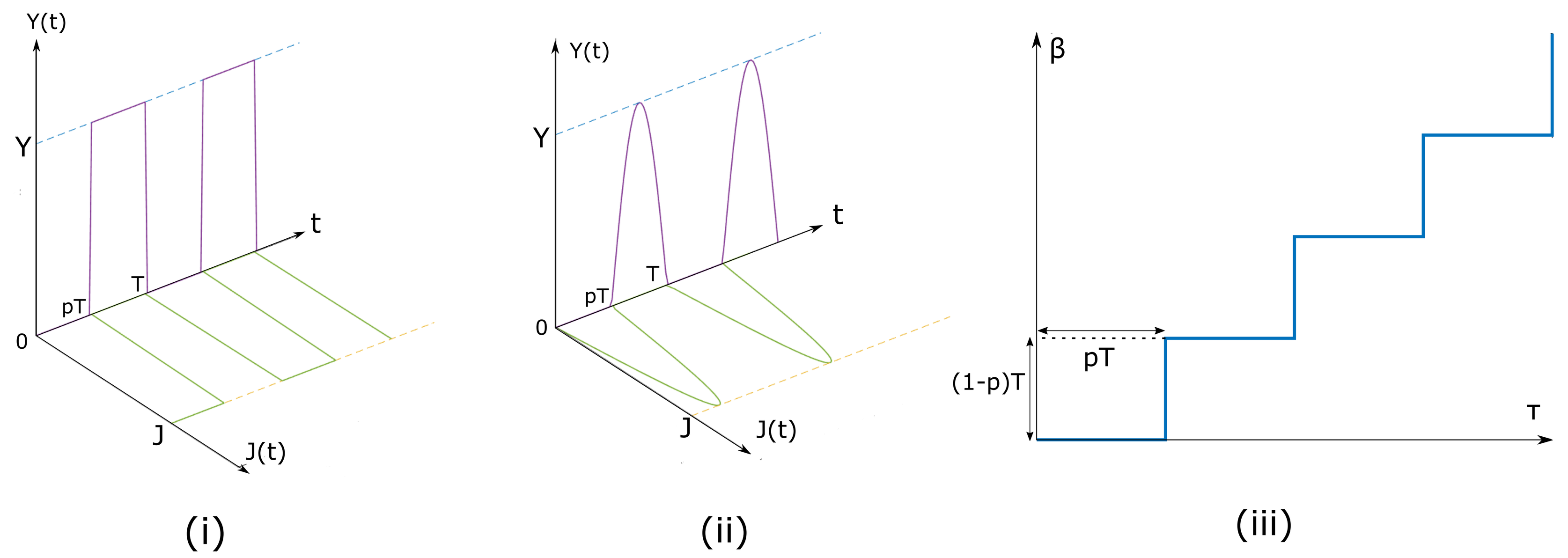


Figure 1. (i) and (ii) are representing the time-dependent Non-Hermitian Floquet model which oscillates between H_1 and H_2 with time-period T.
(iii) Represents the two-dimensional temporal space where, the horizontal axis is the real time unit τ, and the vertical axis is the imaginary time unit β.

τ - Real time unit
β - Imaginary time unit

$$\tau = \frac{pT}{2} \qquad \beta = \frac{(1-p)T}{2} \tag{2}$$

U_τ - Unitary Evolution
U_β - Thermal Evolution

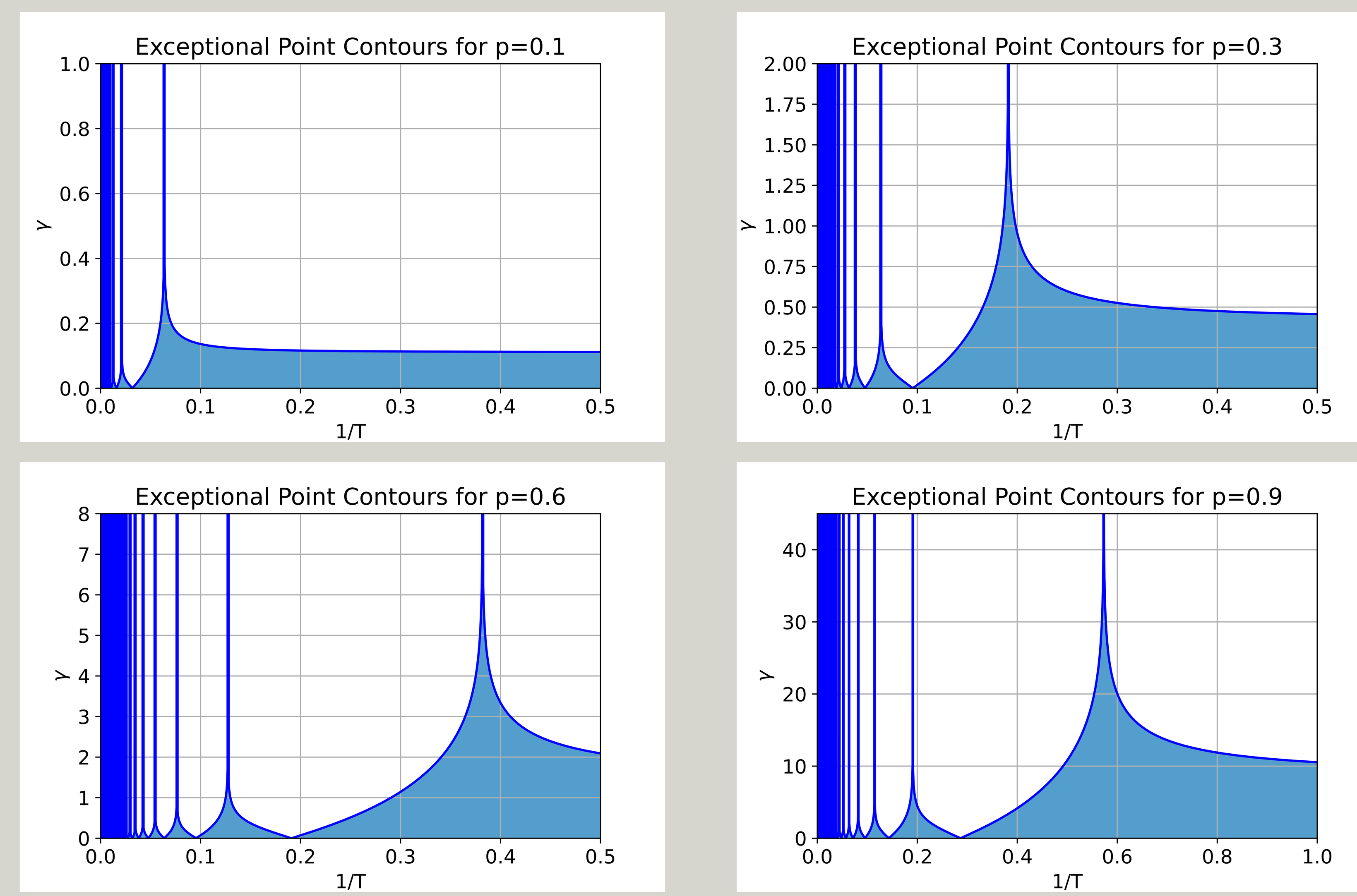
$$U_\tau = \exp\left\{-\iota\sigma_x \int_0^{pT} J(t)dt\right\} \qquad U_\beta = \exp\left\{\sigma_z \int_{pT}^{T} \gamma(t)dt\right\} \tag{3}$$

Numerical Results

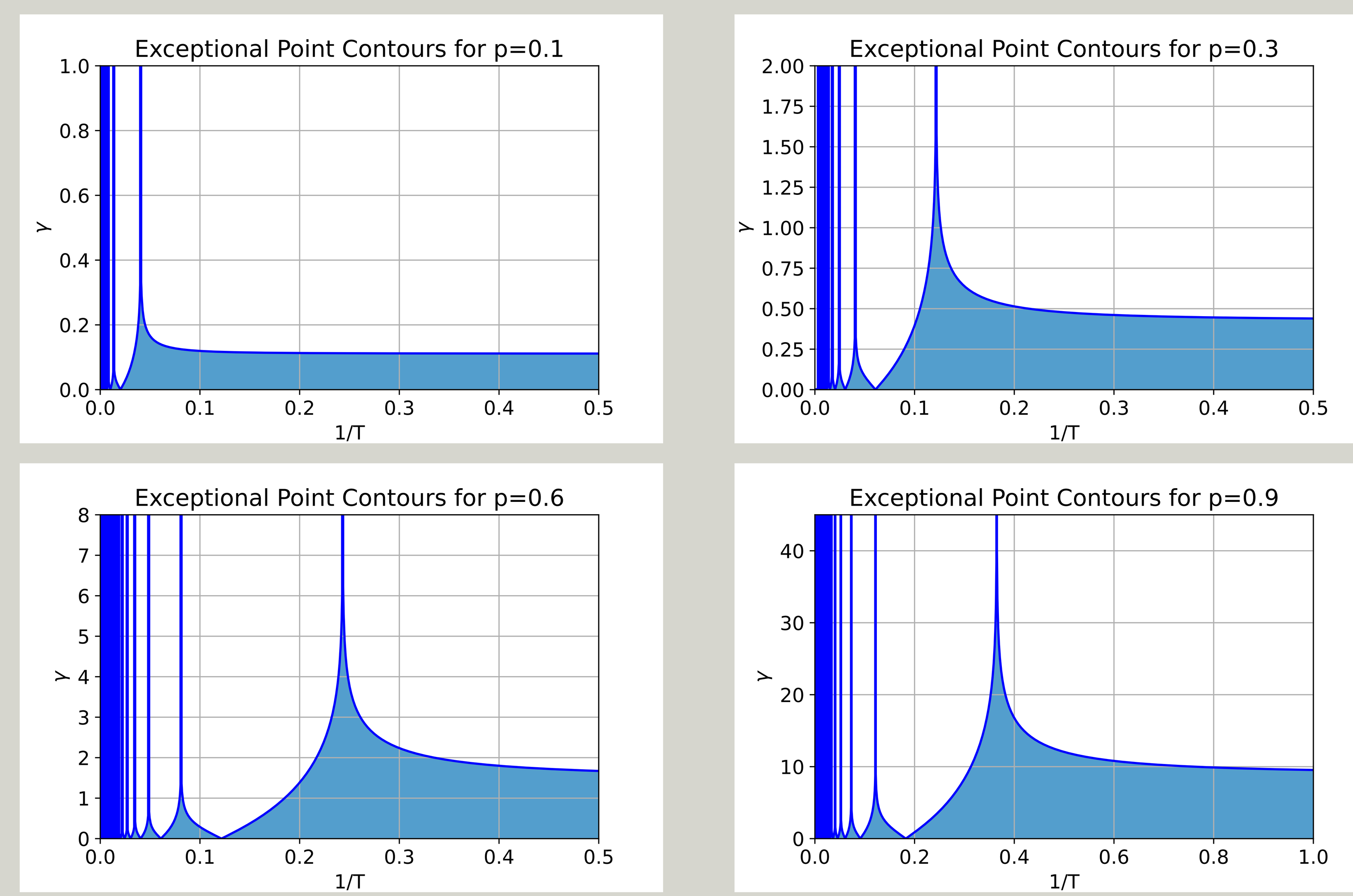
$\mathcal{PT}$-Symmetric region is where the eigenvalues of $U_\beta U_\tau$ are unity.

$$\cosh^2\left(\int_{pT}^{T} \gamma(t)dt\right)\cos^2\left(\int_0^{pT} J(t)dt\right) = 1 \tag{4}$$

Exceptional point contours (Square Wave)



Exceptional point contours (Sinusoidal Wave)



Analytical Results

- The exceptional point contours for the square wave
$$\gamma = \frac{\cosh^{-1}(|\sec(f_a pT)|)}{f_a(1-p)T} \tag{5}$$
- $T \longrightarrow 0^+$
$$\gamma = \frac{p}{1-p} \tag{6}$$
- Resonance indices $\{\gamma(T) \to \infty\}$
$$T = \frac{(2k+1)\pi}{2pf_a} \qquad k \in Z^* \tag{7}$$

Note : f_a is defined as an areal factor, which is **identity** for Square Wave and $\frac{2}{\pi}$ for Sinusoidal Wave.

Effective Floquet Hamiltonian

The effective Floquet Hamiltonian along the EP lines

$$H_F = \frac{1}{T}\tan(2\tau J)\sigma_x + \frac{\iota}{T}\sinh(2\beta\gamma)\sin(2\tau J)\sigma_y + \frac{\iota}{T}\tanh(2\beta\gamma)\sigma_z \tag{8}$$

Evolution for N periods,

$$G(T)^N = \mathbb{1} - \iota N H_F T \equiv E_N(T) \tag{9}$$

Let $|\psi\rangle$ be the initial state, evolving it through N times,

$$\langle\psi| E_N^\dagger(T) E_N(T) |\psi\rangle = \mathbb{1} + (NT)A + (N^2T^2)B \tag{10}$$

$$\boxed{A = \iota\langle\psi| H_F^\dagger - H_F |\psi\rangle}$$

$$\boxed{B = \langle\psi| H_F^\dagger H_F |\psi\rangle}$$

Discussion

- Analysed the time-independent parameters.
- Explored the time-dependent sinusoidal behaviour.
- H_F can be used to study the stroboscopic behaviour.
- These EP degeneracies can be verified experimentally in qubit platforms.

Future Prospects

- What will happen when we couple two qubits where the first qubit is undergoing unitary evolution and the second qubit is going under thermal evolution?
For Example :
$$H = J\mathbb{1}\otimes\sigma_x + \iota\gamma\sigma_z\otimes\mathbb{1} + H_{12} \tag{11}$$
H_{12} is the interaction term.
- Will the dynamics of the system depend on the type of interaction?
- Do we expect higher order exceptional points?

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